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BMN operators with three scalar impurites and the vertex-correlator duality in pp-wave

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Abstract: We calculate 3-point correlation functions of Δ-BMN operators with 3 scalar impurites in $\mathcal{N} = 4$ supersymmetric gauge theory. We use these results to test the pp-wave/SYM duality correspondence of the *vertex-correlator* type. This correspondence relates the coefficients of 3-point correlators of Δ-BMN operators in gauge theory to the 3-string vertex in lightcone string field theory in the pp-wave background. We verify the vertex-correlator duality equation of [6] at the 3 scalar impurites level for supergravity and for string modes.

Keywords: Duality in Gauge Field Theories, Penrose limit and pp-wave background, Supersymmetry and Duality.

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Contents

1. Introduction

This paper continues the study of correlation functions of BMN operators [1]–[6] in the light of the pp-wave/SYM correspondence [7].

The pp-wave/SYM correspondence of Berenstein, Maldacena and Nastase (BMN) [7] represents all massive modes of type IIB superstring in the plane wave background in terms of composite BMN operators of $\mathcal{N} = 4$ Yang-Mills theory in 4D. In its minimal form, this correspondence emphasizes a duality relation between the masses of string states and the anomalous dimensions of the corresponding BMN operators in gauge theory in the large N double scaling limit. This relation has been verified in the planar limit of SYM perturbation theory in [7]–[9]. Calculations in the BMN sector of gauge theory at the nonplanar level were performed in [10, 1, 4, 5] also taking into account mixing effects of planar BMN operators. The minimal mass-dimension type duality relation was extended in [11]–[13] to all orders in the effective genus expansion parameter g_2 and expressed in the form $H_{\text{string}} = H_{\text{SYM}} - J$. Here H_{string} is the full string field theory hamiltonian, and $H_{\text{SYM}} - J = \Delta - J$ is the gauge theory hamiltonian (the conformal dimension) minus the R-charge. Recent work in this direction includes [14].

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In this paper, instead, we address a more ambitious duality relation [6] of a *vertex-correlator* type, summarized in the next section. This type of correspondence for pp-waves was first discussed in [1] and relates the coefficients of 3-point correlators of BMN operators in gauge theory to 3-string vertices in lightcone string field theory in the pp-wave background. It is well-known that in the AdS/CFT scenario, in addition to the relation between the masses of supergravity states and the dimensions of the dual gauge theory operators, one can also compare directly the correlation functions in gauge theory with supergravity interactions in the bulk [15, 16]. Since the pp-wave/CFT correspondence can be viewed as a particular limit of the AdS/CFT correspondence, it is natural to expect that a version of vertex-correlator type duality will hold in the pp-wave/SYM correspondence.

Building on previous work [1, 17, 2, 4], the authors of [6] were able to represent all known gauge theory results for 3-point functions of BMN operators with 2 scalar impurites in terms of a single concise expression involving the 3-string vertex in light-cone string field theory in the pp-wave background. The goal of the present paper is to test this relation at the level of BMN operators with 3 scalar impurites.

In conformal theory, the two- and three-point functions of conformal primary operators are completely determined by conformal invariance of the theory. One can always choose a basis of scalar conformal primary operators such that the two-point functions take the canonical form:

$$\langle \mathcal{O}_I(x)\bar{\mathcal{O}}_J(0)\rangle = \frac{\delta_{IJ}}{(4\pi^2 x^2)^{\Delta_I}}\,, \tag{1.1}$$

and all the nontrivial information of the three-point function is contained in the x-independent coefficient C_{123}:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\bar{\mathcal{O}}_3(0)\rangle = \frac{C_{123}}{(4\pi^2 x_{12}^2)^{\frac{\Delta_1+\Delta_2-\Delta_3}{2}}(4\pi^2 x_1^2)^{\frac{\Delta_1+\Delta_3-\Delta_2}{2}}(4\pi^2 x_2^2)^{\frac{\Delta_2+\Delta_3-\Delta_1}{2}}}\,, \tag{1.2}$$

where $x_{12}^2 := (x_1-x_2)^2$. Since the form of the x-dependence of conformal 3-point functions is universal, it is natural to expect that C_{123} is related to the interaction of the corresponding three string states in the pp-wave background. Note, that in order to be able to use the coefficients C_{123}, it is essential to work on the SYM side with Δ-BMN operators. These operators are defined in such a way that they do not mix with each other (i.e. have definite scaling dimensions Δ) and which are conformal primary operators. Conformal invariance of the $\mathcal{N}=4$ theory then implies that the 2-point correlators of scalar Δ-BMN operators are canonically normalized, and the 3-point functions take the simple form (1.2). Defined in this way, the basis of Δ-BMN operators is unique and distinct from other BMN bases considered in the literature. For 2 scalar impuritites, this Δ-BMN basis was constructed in [4].

In this paper we will work with scalar[1] Δ-BMN operators $\mathcal{O}_{n_1n_2n_3}$ with 3 impurites which correspond to the string bra-state $\langle 0|\alpha^1_{n_1}\alpha^2_{n_2}\alpha^3_{n_3}$. In string theory n_i are the labels of string oscillators $\alpha^i_{n_i}$, and the level matching constraint is $n_1+n_2+n_3=0$. Bare

[1]Vector operators, i.e. Δ-BMN operators with vector impurites, discussed in [18]–[20], will be considered elsewhere.

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single-trace BMN operators with 3 different real scalar impurities [1, 12] are given by

$$
\begin{aligned}
\mathcal{O}_{n_1n_2n_3} &\equiv \mathcal{O}^{123}_{n_1n_2n_3} + \mathcal{O}^{132}_{n_1n_2n_3} \qquad (1.3)\\
&= \frac{1}{J\sqrt{N^{J+3}}} \sum_{0\le l,k}^{l+k\le J} \Big[q_2^l q_3^{l+k} \mathrm{tr}(\phi_1 Z^l \phi_2 Z^k \phi_3 Z^{J-l-k}) + q_3^l q_2^{l+k}\, \mathrm{tr}(\phi_1 Z^l \phi_3 Z^k \phi_2 Z^{J-l-k}) \Big]\,,
\end{aligned}
$$

where $q_2 = e^{2\pi i n_2/J}$ and $q_3 = e^{2\pi i n_3/J}$, and from now on, we always set $n_1 = -n_2 - n_3$. There are two terms on the right hand side of (1.3) since there are two inequivalent orderings of ϕ_1, ϕ_2 and ϕ_3 inside the trace.[2] The pp-wave/SYM duality is supposed to hold in the BMN large N double scaling limit,

$$
J \sim \sqrt{N}\,, \qquad N \to \infty\,. \tag{1.4}
$$

In this limit there remain two free finite dimensionless parameters [7, 10, 1]: the effective coupling constant of the BMN sector of gauge theory,

$$
\lambda' = \frac{g^2_{\mathrm{YM}} N}{J^2} = \frac{1}{(\mu p^+ \alpha')^2} \tag{1.5}
$$

and the effective genus counting parameter

$$
g_2 := \frac{J^2}{N} = 4\pi g_s (\mu p^+ \alpha')^2\,. \tag{1.6}
$$

The right hand sides of (1.5), (1.6) express λ' and g_2 in terms of pp-wave string theory parameters.

Operators (1.3) are the starting point for building the Δ-BMN operators. In interacting field theory, bare operators have to be UV-renormalized and the effects of operator mixing have to be taken into account. It is well-known by now [21, 22, 4, 5] that the single-trace BMN operators mix with the multi-trace operators even in free theory at non-planar level, i.e. starting from order $g_2(\lambda')^0$. Hence, in order to calculate the leading-order contribution to the 3-point coefficient $C_{123} \propto g_2$ in (1.2), one has to work with the order g_2 Δ-BMN operators which involve the single-trace expressions (1.3) plus a linear combination of double-trace operators with coefficients of order $g_2(\lambda')^0$. For the simpler case of 2 scalar impurites, the single-double trace mixing effects have been calculated in [4], and the corresponding conformal 3-point function coefficients C_{123} were determined. One of the main technical results of the present paper will be a determination of the coefficients C_{123} for Δ-BMN operators with 3 scalar impurities (and general oscillator labels $n_2, n_3 \in \mathbb{Z}$).

The paper is organized as follows. In section 2 we summarize the vertex-correlator duality proposal of [6] and write down the relevant equations. In section 3 we calculate 2-point correlators of operators $\mathcal{O}_{n_1n_2n_3}$ to order λ' in planar perturbation theory in the BMN limit. This is necessary in order to canonically normalize the UV-renormalized operators to order λ'. Section 4 contains our main technical results on the field theory side. There we calculate 3-point functions involving operators $\mathcal{O}_{n_1n_2n_3}$. We first derive the conformal

[2] Note that in both terms ϕ_2 in the Z-position l is accompanied by q_2^l, similarly for ϕ_3. Hence, each of the two terms contributes to the same string state.

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expression (1.2) and extract the coefficient C_{123} for 3-point functions containing 1 general and 2 chiral operators (i.e. 1 string and 2 supergravity states in dual string theory). We then generalize this calculation of C_{123} to the case of two string states. In the final section 5 we demonstrate that the results of section 4 are in complete agreement with the vertex-correlator duality prediction [6] of section 2.

2. The vertex-correlator duality

Here we give a brief summary of the duality relation. For more detail, we refer the reader to [6].

For the bosonic external string states $\langle\Phi_i|$ the proposed correspondence relation is

$$\mu(\Delta_1+\Delta_2-\Delta_3)\,C_{123} = \langle\Phi_1|\langle\Phi_2|\langle\Phi_3|\,\mathsf{P}\,\exp\left(\frac{1}{2}\sum_{\mathrm{r,s}=1}^{3}\sum_{m,n=-\infty}^{\infty}\sum_{I=1}^{8}\alpha_m^{\mathrm{r}\,I\dagger}\hat{N}^{\mathrm{rs}}_{mn}\alpha_n^{\mathrm{s}\,I\dagger}\right)|0\rangle_{123}. \tag{2.1}$$

This relation, is conjectured to be valid to all orders in λ' and to the leading order in g_2 in the double scaling limit (1.4). Equation (2.1), originally proposed in [1], is the first key element of the vertex-correlator duality. $\hat{N}_{mn}$ are the Neumann matrices in the α-basis of string oscillators. These matrices were recently calculated in [23] as an expansion in inverse powers of μ at $\mu\to\infty$. Results of [23] for $\hat{N}_{mn}$ constitute the second element of the proposed duality. The relevant for us leading order expressions of $\hat{N}_{mn}$ directly in the α-oscillator basis can be found in the the appendix.

The third and final element of the vertex-correlator duality is the expression [6] for the bosonic part of the string field theory prefactor, P, which appears on the right hand side of (2.1),

$$\mathsf{P} = (-1)^p\, C_{\mathrm{norm}}\,(\mathsf{P}_I+\mathsf{P}_{II})\,, \tag{2.2}$$

where[3]

$$\mathsf{P}_I = \sum_{\mathrm{r}=1}^{3}\sum_{m=-\infty}^{+\infty}\frac{\omega_{\mathrm{r}m}}{\alpha_{\mathrm{r}}}\,\alpha_m^{\mathrm{r}\,I\dagger}\alpha_m^{\mathrm{r}\,I}\,, \tag{2.3}$$

$$\mathsf{P}_{II} = \frac{1}{2}\sum_{\mathrm{r,s}=1}^{3}\sum_{m,n>0}\frac{\omega_{\mathrm{r}m}}{\alpha_{\mathrm{r}}}(\hat{N}^{\mathrm{rs}}_{m-n}-\hat{N}^{\mathrm{rs}}_{mn})\left(\alpha_m^{\mathrm{r}\,I\dagger}\alpha_n^{\mathrm{s}I\dagger}+\alpha_{-m}^{\mathrm{r}\,I\dagger}\alpha_{-n}^{\mathrm{s}I\dagger}-\alpha_m^{\mathrm{r}\,I\dagger}\alpha_{-n}^{\mathrm{s}I\dagger}-\alpha_{-m}^{\mathrm{r}\,I\dagger}\alpha_n^{\mathrm{s}\,I\dagger}\right) \tag{2.4}$$

and

$$C_{\mathrm{norm}} = g_2\frac{\sqrt{y(1-y)}}{\sqrt{J}} = C^{\mathrm{vac}}_{123}\,. \tag{2.5}$$

The only new ingredient here compared to [6] is the overall sign $(-1)^p$ in (2.2), where $p\equiv\frac{1}{2}\sum_{\mathrm{r}=1}^{3}\sum_{m=-\infty}^{+\infty}\alpha_m^{\mathrm{r}\,I\dagger}\alpha_m^{\mathrm{r}\,I}$ counts the number of impurities. For all the cases involving BMN operators with 2 impurities considered in [6], it turns out that $(-1)^p=(-1)^2=1$, and hence is irrelevant. In the present paper, all the cases involving 3 impurities will lead to an overall minus sign, $(-1)^p=(-1)^3=-1$.

[3] We are using standard definitions for the SFT quantities in the pp-wave background such as $\omega_{\mathrm{r}m}$, α_{r} and μ, which are summarized in the appendix.

In terms of the original SFT a-oscillator basis the full prefactor takes a remarkably simple form

$$\mathsf{P} = (-1)^p\, C_{\rm norm} \sum_{\rm r=1}^{3} \left(\sum_{m>0} \frac{\omega_{{\rm r}m}}{\alpha_{\rm r}}\, a_m^{{\rm r}\,I^\dagger} a_m^{{\rm r}\,I} + \mu\, {\rm sign}(\alpha_{\rm r}) a_0^{{\rm r}\,I^\dagger} a_0^{{\rm r}\,I} \right), \tag{2.6}$$

however, as in [6], we will continue using the prefactor in the BMN α-oscillator basis, (2.3) and (2.4), where the comparison with the gauge theory BMN correlators is most direct.

This prefactor, and in particular the second term P_{II}, was constructed in [6] to reproduce a particular class of field theory results for the 3-point functions.[4] It was then successfully tested in [6] against all the available field theory results involving BMN operators with 2 scalar impurites and also the simplest cases involving BMN operators with 3 impurities. In section 5 we will verify that the duality relation (2.1) with the prefactor (2.2) holds at the 3-impurity level.

We emphasize that the matching to field theory results is highly non-trivial even though the choice [6] of the prefactor in (2.2) is "phenomenological". In the next two sections we will assemble a detailed SYM calculation of the 3-point coefficients for BMN correlators with 3 impurities, (4.1), (4.2). This calculation is presented in detail in order to convince the reader that a *coincidental* agreement of our SYM results and the string vertex with the prefactor (2.2) (which a priory knows nothing about 3-impurity operators) is very unlikely. The reader primarily interested in the tests of the correspondence, can skip directly to the final SYM results, eqs. (4.47), (4.48) and (4.53), (4.52) and then to the last section.

3. Two-point correlators

As explained earlier, on the SYM side of our proposed correspondence we must use the Δ-BMN operators $\mathcal{O}$. For BMN operators with 2 scalar impurities this basis was constructed in [4] to order $g_2(\lambda')^0$ and $g_2{}^2(\lambda')^0$ and involves a linear combination of the original single-trace BMN operator and the double-trace (in general multi-trace) BMN operators.

There are two important cases where simplifications occur such that at the leading non-vanishing order in g_2, only the single-trace operators (1.3) need to be taken into account. The first case involves 2-point functions $\langle \mathcal{O}\bar{\mathcal{O}}\rangle$, and will be considered in this section. The second case involves 3-point functions $\langle \mathcal{O}_1\mathcal{O}_2\bar{\mathcal{O}}_3\rangle$, where $\mathcal{O}_1$ and $\mathcal{O}_2$ are chiral BMN operators, and $\mathcal{O}_3$ is a general one, i.e. two supergravity and one string state in dual string theory. This case will be considered in the first part of the next section. It is easy to check (see e.g. [4]) that in both cases the contributions from double- and higher trace operators to $\mathcal{O}$'s give vanishing contributions to the correlators at the leading order in g_2 and in the double-scaling limit (1.4).

[4]A first principles derivation of the string field theory prefactor is highly desirable, but (at least in our view) not yet available inspite of much progress made in the pp-wave lightcone string field theory [23]–[30]. We note that (2.2) is different from the earlier proposals for the prefactor in [25, 27, 23].

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Before we continue we make a final general comment. One can split scalar interactions of the $\mathcal{N}=4$ SYM lagrangian into D-terms, F-terms and K-terms as is done in [1, 4] and show that at one loop level the D-terms cancel against the gluon exchanges and scalar self-energies. So one is left only with F-terms and K-terms. However the K-terms have vanishing contribution in the cases we are going to consider since K-terms couple only to SO(6) traces. Thus, there is only an F-term interaction to consider which has a factor of g_{YM}^2 for every vertex where a ϕ^i line crosses a $Z=\frac{\phi^5+i\phi^6}{\sqrt{2}}$ line, and a factor of $-g_{YM}^2$ when the lines do not cross.

In this section we calculate 2-point correlators $\langle\mathcal{O}\bar{\mathcal{O}}\rangle$ of renormalized operators (1.3) in planar perturbation theory to order λ'. This is needed to normalize the operators correctly, such that (1.1) holds at order λ'. In this and the next section we will be calculating Feynman diagrams in dimensional reduction to $4-2\epsilon$ dimensions, and in coordinate space. Our calculations follow and generalize the approach of [2].

We note that bare operators in (1.3) were normalized in such a way that their free 2-point planar correlator is

$$\langle\bar{\mathcal{O}}_{n_1n_2n_3}(0)\mathcal{O}_{\tilde{n}_1\tilde{n}_2\tilde{n}_3}(x)\rangle=\delta_{n_2,\tilde{n}_2}\delta_{n_3,\tilde{n}_3}\Delta(x)^{J+3} \tag{3.1}$$

in the BMN limit (1.4). Here $\Delta(x)$ is the scalar propagator,

$$\Delta(x)=\frac{\Gamma(1-\epsilon)}{(4\pi^{2-\epsilon})(x^2)^{1-\epsilon}}\,. \tag{3.2}$$

There are four contributions to consider, $\langle\bar{\mathcal{O}}^{123}(0)\mathcal{O}^{123}(x)\rangle$, $\langle\bar{\mathcal{O}}^{132}(0)\mathcal{O}^{132}(x)\rangle$, $\langle\bar{\mathcal{O}}^{123}(0)$ $\mathcal{O}^{132}(x)\rangle$ and $\langle\bar{\mathcal{O}}^{132}(0)\mathcal{O}^{123}(x)\rangle$. The last two correlators vanish in free theory (since the 3 ϕ's are different), and will be shown to vanish also at order λ' at the planar level.

We first calculate the interacting part of $\langle\bar{\mathcal{O}}^{123}(0)\mathcal{O}^{123}(x)\rangle$,

$$\langle\bar{\mathcal{O}}^{123}(0)\mathcal{O}^{123}(x)\rangle=\frac{\Delta(x)^{J+3}}{J^2}(-g_{YM}^2N)\,I(x)\,(P_1+P_2+P_3)\,, \tag{3.3}$$

where $I(x)$ is the interaction integral with $\Delta(x)^2$ removed:

$$\begin{aligned}I(x)&=\left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^2(x^2)^{2-2\epsilon}\int\frac{d^{4-2\epsilon}y}{(y^2)^{2-2\epsilon}(y-x)^{2(2-2\epsilon)}}\\&=\frac{1}{8\pi^2}\left(\frac{1}{\epsilon}+\gamma+1+\log\pi+\log x^2+O(\epsilon)\right).\end{aligned} \tag{3.4}$$

We will use a subtraction scheme which subtracts the $1/\epsilon$ pole together with (an arbitrary) finite part s

$$\frac{1}{\epsilon}+s\,. \tag{3.5}$$

P_1, P_2 and P_3 on the right hand side of (3.3) are the total contributions of the phase factors for the diagrams of figure 1, figure 2 and figure 3 respectively. Denoting by q_2, q_3 the BMN phases of the $\mathcal{O}_{\tilde{n}_1\tilde{n}_2\tilde{n}_3}(x)$ operator, and by $\bar{\mathrm{r}}_2,\bar{\mathrm{r}}_3$ the BMN phases of the $\bar{\mathcal{O}}_{n_1n_2n_3}(0)$ operator,

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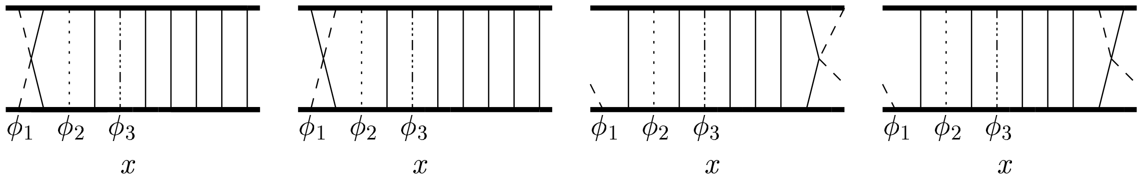

Figure 1: Interacting diagrams for the 2-point function where ϕ_1 interacts with Z. These diagrams give rise to P_1.

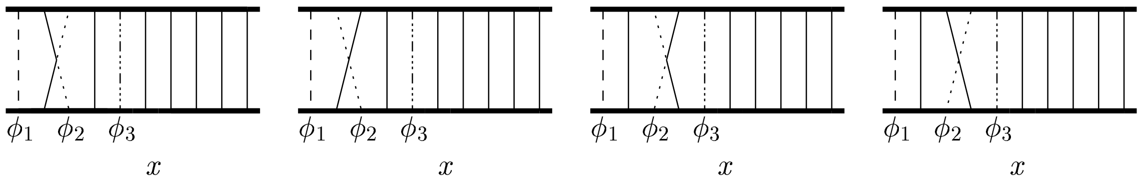

Figure 2: Interacting diagrams for the 2-point function with ϕ_2 interactions. These diagrams give rise to P_2.

we obtain

$$\begin{aligned}
P_1 &= \sum_{1\leq l, 0\leq k}^{l+k\leq J} [q_2^l q_3^{l+k} \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^{l+k} - q_2^l q_3^{l+k} \bar{\mathrm{r}}_2^{l-1} \bar{\mathrm{r}}_3^{l+k-1}] + \\
&\quad + \sum_{0\leq l, 0\leq k}^{l+k\leq J-1} [q_2^l q_3^{l+k} \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^{l+k} - q_2^l q_3^{l+k} \bar{\mathrm{r}}_2^{l+1} \bar{\mathrm{r}}_3^{l+k+1}] \\
&= \sum_{0\leq l, 0\leq k}^{l+k\leq J-1} [q_2^{l+1} q_3^{l+k+1} \bar{\mathrm{r}}_2^{l+1} \bar{\mathrm{r}}_3^{l+k+1} - q_2^{l+1} q_3^{l+k+1} \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^{l+k}] + \\
&\quad + \sum_{0\leq l, 0\leq k}^{l+k\leq J-1} [q_2^l q_3^{l+k} \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^{l+k} - q_2^l q_3^{l+k} \bar{\mathrm{r}}_2^{l+1} \bar{\mathrm{r}}_3^{l+k+1}] \\
&= \sum_{0\leq l, 0\leq k}^{l+k\leq J-1} q_2^l q_3^{l+k} \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^{l+k} (1 + q_2 q_3 \bar{\mathrm{r}}_2 \bar{\mathrm{r}}_3 - q_2 q_3 - \bar{\mathrm{r}}_2 \bar{\mathrm{r}}_3) \\
&= \sum_{0\leq l, 0\leq k}^{l+k\leq J-1} (q_2 \bar{\mathrm{r}}_2)^l (q_3 \bar{\mathrm{r}}_3)^{l+k} [(1 - q_2 q_3)(1 - \bar{\mathrm{r}}_2 \bar{\mathrm{r}}_3)] \,. \qquad (3.6)
\end{aligned}$$

To derive (3.6) we have added the contributions of four diagrams in figure 1 and noted that contributions of diagrams where a Z line crosses a ϕ line in the Z-ϕ interaction (the second and the fourth diagrams in figure 1) have a relative minus sign compared to the Z-ϕ interaction without crossing (the first and the third diagrams in figure 1).

Similarly from four diagrams of figure 2 and from four diagrams of figure 3 we get

$$P_2 = \sum_{0\leq l, 0\leq k}^{l+k\leq J-1} (q_2 \bar{\mathrm{r}}_2)^l (q_3 \bar{\mathrm{r}}_3)^{l+k} [(1 - q_2)(1 - \bar{\mathrm{r}}_2)] q_3 \bar{\mathrm{r}}_3$$

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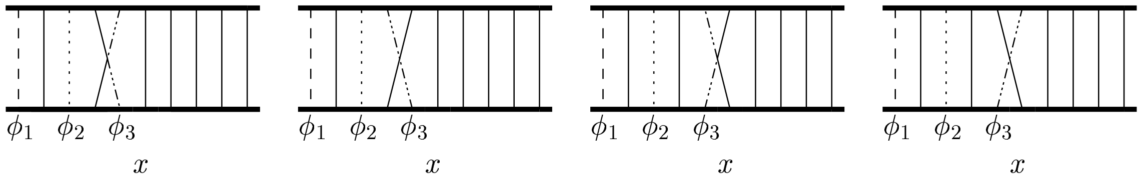

Figure 3: Interacting diagrams for the 2-point function with ϕ_3 interactions. These diagrams give rise to P_3.

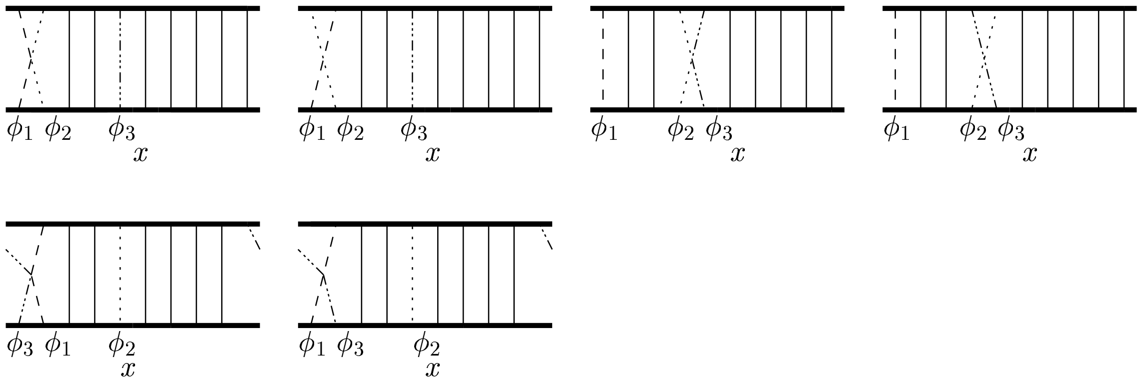

Figure 4: Interacting diagrams for the two-point function. These diagrams give rise to $P_4 = 0$. There are six additional diagrams with ϕ_2 and ϕ_3 exchanged. Their sum is also zero.

$$P_3 = \sum_{0\le l,0\le k}^{l+k\le J-1} (q_2\bar{\mathrm{r}}_2)^l (q_3\bar{\mathrm{r}}_3)^{l+k} [(1-q_3)(1-\bar{\mathrm{r}}_3)]\,. \tag{3.7}$$

We now evaluate the double sum:

$$\begin{aligned}\sum_{0\le l,0\le k}^{l+k\le J-1} (q_2\bar{\mathrm{r}}_2)^l q_3\bar{\mathrm{r}}_3)^{l+k} &= \sum_{l=0}^{J-1} (q_2\bar{\mathrm{r}}_2 q_3\bar{\mathrm{r}}_3)^l \sum_{k=0}^{J-1-l} (q_3\bar{\mathrm{r}}_3)^k \\ &= \begin{cases} 0 & \text{when } q_2\bar{\mathrm{r}}_2 \neq 1 \text{ and } q_3\bar{\mathrm{r}}_3 \neq 1 \\ J(J+1)/2 & \text{when } q_2\bar{\mathrm{r}}_2 = 1 = q_3\bar{\mathrm{r}}_3 \\ -\frac{J}{q_2\bar{\mathrm{r}}_2 - 1} & \text{when } q_2\bar{\mathrm{r}}_2 \neq 1 \text{ and } q_3\bar{\mathrm{r}}_3 = 1 \\ \frac{J}{q_3\bar{\mathrm{r}}_3 - 1} & \text{when } q_2\bar{\mathrm{r}}_2 = 1 \text{ and } q_3\bar{\mathrm{r}}_3 \neq 1 \end{cases}\end{aligned}$$

It is clear that, in the BMN limit (1.4), the correlator is non-zero only when $q_2\bar{\mathrm{r}}_2 = 1 = q_3\bar{\mathrm{r}}_2$, that is, when the operators in the correlator are the same.

The result for the second correlator, $\langle\bar{\mathcal{O}}^{132}(0)\mathcal{O}^{132}(x)\rangle$, is obtained from the first one by interchanging labels 2 and 3. The sum of the two contributions, $\langle\bar{\mathcal{O}}^{123}(0)\mathcal{O}^{123}(x)\rangle + \langle\bar{\mathcal{O}}^{132}(0)\mathcal{O}^{132}(x)\rangle$, will have the second term on the right hand side of (3.8) doubled up, and the third and fourth terms cancelled.

We now show that the other two correlators $\langle\bar{\mathcal{O}}^{123}(0)\mathcal{O}^{132}(x)\rangle$ and $\langle\bar{\mathcal{O}}^{132}(0)\mathcal{O}^{123}(x)\rangle$, vanish in our case. There are 12 diagrams to consider, the first 6 are shown in figure 4.

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The combined phase factor with these six diagrams is:

$$
\begin{aligned}
P_4 &= \sum_{k=0}^{J} \left(q_2^0 q_3^k \bar{\mathrm{r}}_2^0 \bar{\mathrm{r}}_3^k - q_2^0 q_3^k \bar{\mathrm{r}}_2^J \bar{\mathrm{r}}_3^k \right) + \sum_{l=0}^{J} \left(q_2^l q_3^l \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^l \right) - q_2^l q_3^l \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^l) + \sum_{l=0}^{J} \left(q_2^l q_3^J \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^J - q_2^l q_3^0 \bar{\mathrm{r}}_2^l \bar{\mathrm{r}}_3^J \right) \\
&= 0\,.
\end{aligned} \tag{3.8}
$$

The remaining six diagrams are obtained from the ones in figure 4 by exchanging ϕ_2 and ϕ_3. They also sum to zero. Thus, the non-diagonal terms do not contribute to the correlator at the planar level.

Finally, combining with the free result we obtain

$$
\begin{aligned}
\langle \bar{\mathcal{O}}_{n_1 n_2 n_3}(0) \mathcal{O}_{\tilde{n}_1 \tilde{n}_2 \tilde{n}_3}(x) \rangle \;=\; & \delta_{n_2 \tilde{n}_2} \delta_{n_3 \tilde{n}_3} \Delta(x)^{J+3} I(x) \times \\
& \times \{ 1 - g_{YM}^2 N [(1 - q_2 q_3)(1 - \bar{q}_2 \bar{q}_3) + (1 - q_2)(1 - \bar{q}_2) + \\
& + (1 - \bar{q}_3)(1 - q_3)] \}\,,
\end{aligned}
$$

where the four terms in curly brackets correspond respectively to the free contribution, P_1, P_2 and P_3. Now, substituting (3.4) for $I(x)$ with a subtraction (3.5), and using an expansion

$$
\Delta(x)^\alpha \simeq 1 + \alpha \log \Delta(x) = 1 + \alpha(-\log 4\pi^2 - \log x^2) + O(\epsilon)) \tag{3.9}
$$

we derive the final expression for the 2-point function,

$$
\langle \bar{\mathcal{O}}_{n_1 n_2 n_3}(0) \mathcal{O}_{\tilde{n}_1 \tilde{n}_2 \tilde{n}_3}(x) \rangle = \delta_{n_2 \tilde{n}_2} \delta_{n_3, \tilde{n}_3} \Delta(x)^{J+3+\alpha} [1 - \alpha(\gamma + 1 - \log 4\pi - s)]\,, \tag{3.10}
$$

where α denotes

$$
\alpha = \frac{\lambda'}{2} [(n_2 + n_3)^2 + n_2^2 + n_2^2]\,, \tag{3.11}
$$

and, from the right hand side of (3.10), it has to be identified with the anomalous dimension of $\mathcal{O}_{n_1 n_2 n_3}$,

$$
\Delta - J = 3 + \alpha = 3 + \frac{\lambda'}{2} [(n_2 + n_3)^2 + n_2^2 + n_2^2]\,, \tag{3.12}
$$

in agreement with dual string theory prediction.

The normalized operator is given by

$$
\begin{aligned}
\mathcal{O}_{n_1 n_2 n_3} \;=\; \frac{1 + \alpha/2(\gamma + 1 - \log 4\pi - s)}{J\sqrt{N^{J+3}}} \sum_{0 \le l,k}^{l+k \le J} & [q_2^l q_3^{l+k} \,\mathrm{tr}(\phi_1 Z^l \phi_2 Z^k \phi_3 Z^{J-l-k}) + \\
& + q_3^l q_2^{l+k} \,\mathrm{tr}(\phi_1 Z^l \phi_3 Z^k \phi_2 Z^{J-l-k})]\,.
\end{aligned} \tag{3.13}
$$

4. Three-point functions

Here our goal is evaluate 3-point functions involving BMN operators with 3 scalar impurities in planar perturbation theory to order λ'. We will consider two such 3-point functions,

$$
G_3(x_1, x_2) = \langle \bar{\mathcal{O}}^J_{n_1 n_2 n_3}(0) \mathcal{O}^{J_1}_{n_1' n_2' n_3'}(x_1) \mathcal{O}^{J_2}_{\mathrm{vac}}(x_2) \rangle \tag{4.1}
$$

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and

$$G'_3(x_1,x_2) = \langle \bar{\mathcal{O}}^J_{n_1n_2n_3}(0)\mathcal{O}^{J_1}_{n'_2-n'_2}(x_1)\mathcal{O}^{J_2}_0(x_2)\rangle\,. \tag{4.2}$$

Here $\mathcal{O}^J_{n_1n_2n_3}$ with $n_1 = -n_2 - n_3$, is a Δ-BMN operator with 3 scalar impurities, it is given by (3.13) plus multi-trace expressions[5] at higher orders in g_2. The operator $\mathcal{O}^{J_1}_{n'_2-n'_2}$ is a Δ-BMN operator with 2 scalar impurities, at the classical single-trace level it is given by

$$\mathcal{O}^{J_1}_{n'_2-n'_2} = \frac{1}{\sqrt{JN^{J_1+2}}}\sum_{l=0}^{J_1} r_2^l\,\mathrm{tr}(\phi_1 Z^l \phi_2 Z^{J_1-l})\,, \tag{4.3}$$

and the remaining operators are chiral and are protected against quantum corrections,

$$\mathcal{O}^{J_2}_0 = \frac{1}{\sqrt{N^{J_2+1}}}\,\mathrm{tr}(\phi_3 Z^{J_2})\ ,\quad \mathcal{O}^{J_2}_{\mathrm{vac}} = \frac{1}{\sqrt{J_2 N^{J_2}}}\,\mathrm{tr}(Z^{J_2})\,. \tag{4.4}$$

We also note that the R-charge conservation implies that

$$J_2 = J - J_1\,. \tag{4.5}$$

In the first subsection we will derive conformal expressions (1.2) and determine the 3-point coefficients C_{123} for both of these Green functions in the settings where the barred operator is general, and the two unbarred operators are chiral, i.e. correspond to two supergravity states. As mentioned earlier, in this case, only the single-trace contributions to the operators are relevant at the leading non-vanishing order in g_2, thus simplifying our analysis significantly.

In the second subsection we will calculate the 3-point coefficients of (4.1) and (4.2) in the general case of two non-chiral operators. Here the double-trace corrections are important, in order to derive the conformal expression (1.2). However, using a simple trick we will show how to uniquely determine the coefficients C_{123} directly from the single-trace expressions for the operators, thus obtaining the main results of this section, eqs. (4.47), (4.48) and (4.53), (4.52).

4.1 One string and two supergravity states

As explained above, in this subsection only, we set $n'_1 = n'_2 = n'_3 = 0$ and consider

$$G_3(x_1,x_2) = \langle \bar{\mathcal{O}}^J_{n_1n_2n_3}(0)\mathcal{O}^{J_1}_{000}(x_1)\mathcal{O}^{J_2}_{\mathrm{vac}}(x_2)\rangle \tag{4.6}$$

and

$$G'_3(x_1,x_2) = \langle \bar{\mathcal{O}}^J_{n_1n_2n_3}(0)\mathcal{O}^{J_1}_{00}(x_1)\mathcal{O}^{J_2}_0(x_2)\rangle\,. \tag{4.7}$$

At first we consider the 3-point function $G_3(x_1,x_2)$ and express it as follows:

$$G_3(x_1,x_2) = \frac{1+\frac{\alpha}{2}(\gamma+1-\log 4\pi - s)}{J\sqrt{N^{J+3}}}\frac{N^{J+2}J_2}{J_1\sqrt{J_2N^{J_1+3}N^{J_2}}}\times$$

$$\times\Delta(x_1)^{J_1+3}\Delta(x_2)^{J_2}\left(X - \lambda Y K(x_1,x_2)\right), \tag{4.8}$$

[5] In fact, it follows from the analysis of [4] that at the relevant to us first order in g_2, only the double-trace corrections and only to the barred operators in (4.1) and (4.2) give non-vanishing contributions.

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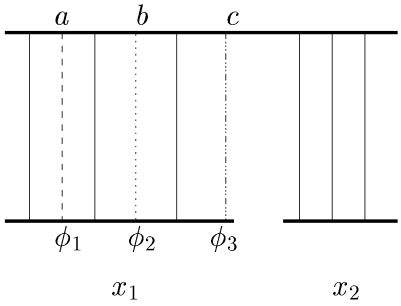

Figure 5: A typical free diagram for G_3. This diagram gives rise to X_{123}. There are five additional diagrams with ϕ_1, ϕ_2 and ϕ_3 interchanged. a, b and c are the positions of the impurities in the trace. For this particular diagram, $a = 2$, $b = 4$, $c = 6$.

where $\lambda = g_{YM}^2 N$ and X and Y are the combined phase-factors at the free and interacting level respectively. $K(x_1, x_2)$ is the interaction integral for the diagrams depicted on figures 6–9 (as in [2]):

$$
\begin{aligned}
K(x_1,x_2) &= \left(\frac{\Gamma(1-\epsilon)}{4\pi^{2-\epsilon}}\right)^2 (x_1^2)^{1-\epsilon}(x_2^2)^{1-\epsilon} \int \frac{d^{4-2\epsilon}y}{(y^2)^{2-2\epsilon}(y-x_1)^{2(1-\epsilon)}(y-x_2)^{2(1-\epsilon)}} \\
&= \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \gamma + 2 + \log\pi + \log\frac{x_1^2 x_2^2}{x_{12}^2} + O(\epsilon)\right). \qquad (4.9)
\end{aligned}
$$

The first fraction on the right hand side of (4.8) arises from the normalization of $\bar{\mathcal{O}}^J_{n_1 n_2 n_3}$. The denominator of the second fraction arises from the normalizations of the other two operators, while the summation over the $J+2$ loops gives a factor of N^{J+2}. The remaining factor of J_2 comes from the Wick contractions with $\mathcal{O}^{J_2}_{\text{vac}}$.

Free diagrams are shown in figure 5. There are six diagrams because of the six different ways of arranging three ϕ's in a trace. We denote with X_{123} the combined phase factor of a free diagram where ϕ_1 comes first, ϕ_2 is second and ϕ_3 is third.

We have:

$$
\begin{aligned}
X_{123} &= \sum_{a=1}^{J_1+1}\sum_{b=a+1}^{J_1+2}\sum_{c=b+1}^{J_1+3} \bar{q}_2^{\,b-a-1}\bar{q}_3^{\,c-a-2} \overset{J\to\infty}{\longrightarrow} \\
&\longrightarrow J^3 \int_0^{J_1/J} da \int_a^{J_1/J} db \int_b^{J_1/J} dc\, e^{-2\pi i n_2(b-a)} e^{-2\pi i n_3(c-a)} \qquad (4.10) \\
X_{231} &= \sum_{a=1}^{J_1+1}\sum_{b=a+1}^{J_1+2}\sum_{c=b+1}^{J_1+3} \bar{q}_2^{\,J-(c-a-2)}\bar{q}_3^{\,J-(c-b-1)} \overset{J\to\infty}{\longrightarrow} \\
&\longrightarrow J^3 \int_0^{J_1/J} da \int_a^{J_1/J} db \int_c^{J_1/J} dc\, e^{-2\pi i n_2(-1)(c-a)} e^{-2\pi i n_3(-1)(c-b)} \qquad (4.11) \\
X_{312} &= \sum_{a=1}^{J_1+1}\sum_{b=a+1}^{J_1+2}\sum_{c=b+1}^{J_1+3} \bar{q}_2^{\,c-b-1}\bar{q}_3^{\,J-(b-a-1)} \overset{J\to\infty}{\longrightarrow} \\
&\longrightarrow J^3 \int_0^{J_1/J} da \int_a^{J_1/J} db \int_b^{J_1/J} dc\, e^{-2\pi i n_2(c-b)} e^{-2\pi i n_3(-1)(b-a)}, \qquad (4.12)
\end{aligned}
$$

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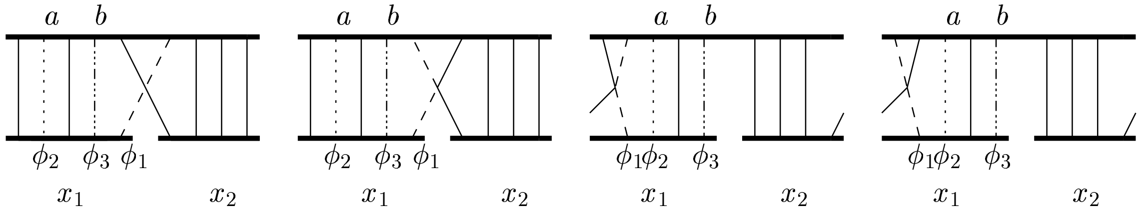

Figure 6: Interacting diagrams for G_3. These diagrams give rise to Y_{123}. There are four additional diagrams with ϕ_2 and ϕ_3 exchanged.

where a, b, c are the positions of the first, second and third impurities in the trace. Also it is easy to see that X_{132} is equal to X_{123} with $\bar{q}_2$ and $\bar{q}_3$ exchanged, X_{213} is X_{312} with $\bar{q}_2$ and $\bar{q}_3$ exchanged and, X_{321} is X_{231} with $\bar{q}_2$ and $\bar{q}_3$ exchanged.

The sum of the six X's is:

$$X = J^3 \int_0^{J_1/J} da \int_0^{J_1/J} db \int_0^{J_1/J} dc\, e^{-2\pi i n_2 (b-a)} e^{-2\pi i n_3 (c-a)} . \tag{4.13}$$

For example the part of the above sum with $c > b > a$ is X_{123}, the part with $c > a > b$ is X_{213} and so on. Evaluating the integral we get

$$X = J^3 2^3 \frac{\sin(\pi n_2 J_1/J) \sin(\pi n_3 J_1/J) \sin(\pi(n_2+n_3)J_1/J)}{(2\pi)^3 (n_2+n_3) n_2 n_3} . \tag{4.14}$$

We now calculate the phase factors coming from interacting planar diagrams. In the case where ϕ_1 interacts with Z we have eight diagrams with the first four shown in figure 6 and the remaining four obtained by interchanging ϕ_2 and ϕ_3. We do not need to consider diagrams where ϕ_i interacts with ϕ_j since they will be suppressed in the BMN limit (1.4) relative to ϕ-Z interactions of figure 6.

The phase combined factor of the four diagrams in figure 6 is:

$$\begin{aligned} Y_{123} &= \sum_{a=1}^{J_1+1} \sum_{b=a+1}^{J_1+2} \left[-\bar{q}_2^{J-(J_1+3-a-1)} \bar{q}_3^{J-(J_1+3-b)} + \bar{q}_2^{J-(J_1+3-a-2)} \bar{q}_3^{J-(J_1+3-b-1)} \right] + \\ &\quad + \sum_{a=3}^{J_1+2} \sum_{b=a+1}^{J_1+3} \left[\bar{q}_2^{a-3} \bar{q}_3^{b-4} - \bar{q}_2^{a-2} \bar{q}_3^{b-3} \right] \\ &= \sum_{a=1}^{J_1+1} \sum_{b=a+1}^{J_1+2} \bar{q}_2^{-(J_1-a+1)} \bar{q}_3^{-(J_1-b+2)} (1 - \bar{q}_2^{-1} \bar{q}_3^{-1}) + \sum_{a=3}^{J_1+2} \sum_{b=a+1}^{J_1+3} \bar{q}_2^{a-3} \bar{q}_3^{b-4} (1 - \bar{q}_2 \bar{q}_3) \rightarrow \\ &\quad \rightarrow \frac{2\pi i (n_2+n_3)}{J} \left[\sum_{a=3}^{J_1+2} \sum_{b=a+1}^{J_1+3} \bar{q}_2^{a-3} \bar{q}_3^{b-4} - \sum_{a=1}^{J_1+1} \sum_{b=a+1}^{J_1+2} \bar{q}_2^{a-1} \bar{q}_3^{b-2} \bar{q}_2^{-J_1} \bar{q}_3^{-J_1} \right] . \end{aligned} \tag{4.15}$$

In deriving the last line above, we have used that $1 - \bar{q}_2 \bar{q}_3 \rightarrow \frac{2\pi i (n_2+n_3)}{J}$ in the BMN limit. Converting the sum into an integral we get

$$\begin{aligned} Y_{123} &= 2\pi i (n_2+n_3) J \int_0^{J_1/J} da \int_a^{J_1/J} db\, e^{-2\pi i n_2 a} e^{-2\pi i n_3 b} [1 - e^{-2\pi i (n_3+n_2)(-J_1/J)}] \\ &= -J[1 - e^{-(A_2+A_3)J_1/J}] \frac{e^{(A_3+A_2)J_1/J} A_3 - e^{A_3 J_1/J}(A_2+A_3) + A_2}{A_3 A_2} , \end{aligned} \tag{4.16}$$

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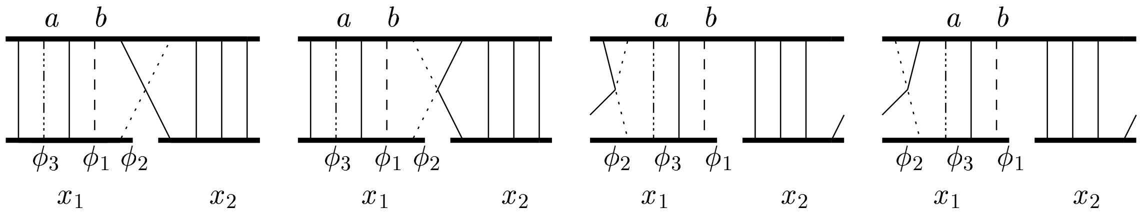

Figure 7: Interacting diagrams for G_3. These diagrams give rise to Y_{231}. There are four additional diagrams with ϕ_1 and ϕ_3 exchanged.

where $A_2 = -2\pi i n_2$ and $A_3 = -2\pi i n_3$. We must add 4 more diagrams with ϕ_2 and ϕ_3 exchanged. This gives the expression above with n_2 and n_3 exchanged. If we sum the two contributions we get:

$$Y_{123} + Y_{132} = \frac{4(n_3+n_2)^2 J \sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{\pi(n_3+n_2)n_2 n_3}. \tag{4.17}$$

In the case where ϕ_2 interacts we have again eight diagrams, see figure 7.

The combined phase factor of four diagrams in figure 7 is easily obtained by substituting $n_2 \to n_3$ and $n_3 \to -(n_2+n_3)$ into (4.16), as follows from comparing diagrams in figures 6 and 7 and remembering that $n_1 := -(n_2+n_3)$. In the BMN limit we have,

$$Y_{231} = J[1-e^{A_2 J_1/J}]\frac{-e^{-A_2 J_1/J}(A_2+A_3) + e^{-(A_3+A_2)J_1/J}A_2 + A_3}{A_3(A_2+A_3)}. \tag{4.18}$$

Now we consider the above diagrams with ϕ_1 and ϕ_3 exchanged, i.e. $-(n_2+n_3) \leftrightarrow n_3$ and n_2 unchanged in (4.18),

$$Y_{213} = J[1-e^{A_2 J_1/J}]\frac{e^{-A_2 J_1/J}A_3 + e^{A_3 J_1/J}A_2 - A_2 - A_3}{A_3(A_2+A_3)}. \tag{4.19}$$

Summing the above contributions we arrive at

$$Y_{231} + Y_{213} = \frac{4n_2^2 J \sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{\pi(n_3+n_2)n_2 n_3}. \tag{4.20}$$

Similarly, for diagrams in figure 8 and their four partners, we find:

$$Y_{312} + Y_{321} = \frac{4n_3^2 J \sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{\pi(n_3+n_2)n_2 n_3}. \tag{4.21}$$

For completeness, we note that diagrams without x_1-to-x_2 connection, depicted in figure 9, sum to zero.

Putting together all the expressions above, we get for G_3

$$G_3(x_1,x_2) = \frac{1+\alpha/2(\gamma+1-\log 4\pi - s)}{J\sqrt{N^{J+3}}}\frac{N^{J+2}J_2}{J_1\sqrt{J_2 N^{J_1+3}N^{J_2}}}\Delta(x_1)^{J_1+3}\Delta(x_2)^{J_2}\times$$

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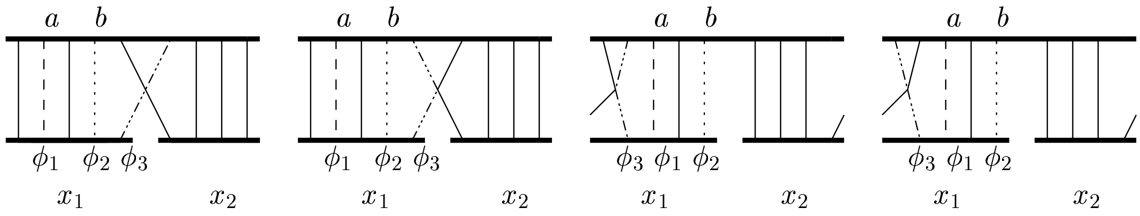

Figure 8: Interacting diagrams for G_3, contributing to Y_{312}. There are four additional diagrams with ϕ_1 and ϕ_2 exchanged.

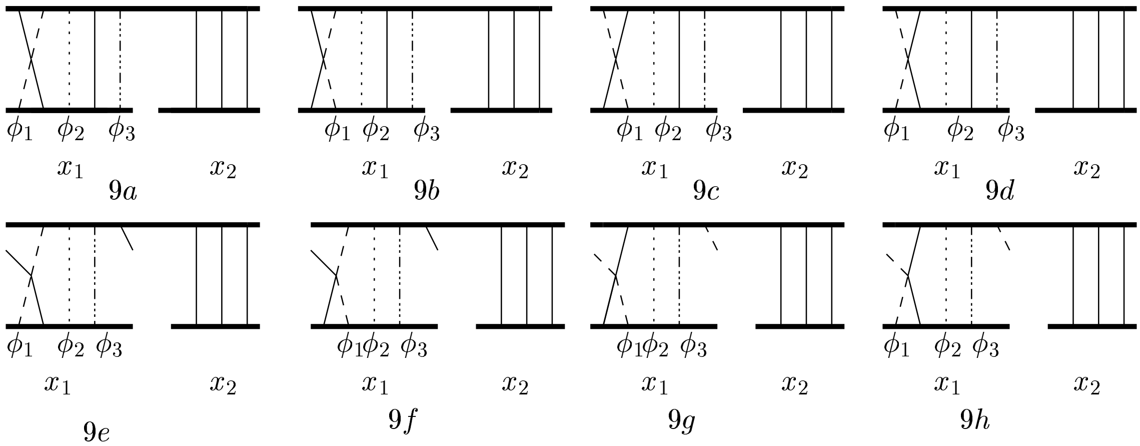

Figure 9: Interacting diagrams for G_3. Diagrams $9a, 9c, 9e$ and $9g$ have an opposite sign with respect to $9b, 9d, 9f$ and $9h$, so they cancel pairwise. There are additional diagrams where ϕ_2 and ϕ_3 are exchanged which also add up zero. Additional diagrams where ϕ_2 or ϕ_3 (rather than ϕ_1 interact with Z also cancel in the same way.

$$
\begin{aligned}
&\times \frac{J^3}{\pi^3} \frac{\sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{(n_3+n_2)n_2 n_3} \times \\
&\times \left[1 - \frac{\lambda'}{4}((n_3+n_2)^2 + n_2^2 + n_3^2)) \left(\frac{1}{\epsilon} + \gamma + 2 + \log\pi + \log\frac{x_1^2 x_2^2}{x_{12}^2}\right)\right]. \quad (4.22)
\end{aligned}
$$

Subtracting $1/\epsilon + s$ we obtain the result

$$
G_3(x_1, x_2) = C_{123}\ \Delta(x_1)^{J_1+3+\alpha/2}\Delta(x_2)^{J_2+\alpha/2}\Delta(x_{12})^{-\alpha/2} \quad (4.23)
$$

with

$$
C_{123} = \frac{J^2\sqrt{J_2}}{J_1 N \pi^3} \frac{\sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{(n_3+n_2)n_2 n_3} \left(1 - \frac{\alpha}{2}\right). \quad (4.24)
$$

A few comments are in order. First, we note that we have proved that to order in λ' and g_2 we are working here, G_3 takes the conformal form of (1.2). This is so since (4.23) is nothing other than the conformal expression (1.2) for G_3. Second, we have derived the expression for the coefficient C_{123} given by (4.24). This expression does not depend on s and, hence, is the subtraction scheme independent, as expected. In what follows, and in parallel with [6, 4], we will use only the leading order in λ' part of C_{123}, i.e. will set $\alpha = 0$ in (4.24). This is because the, yet unaccounted, mixing effects at order λ' can change *constant*

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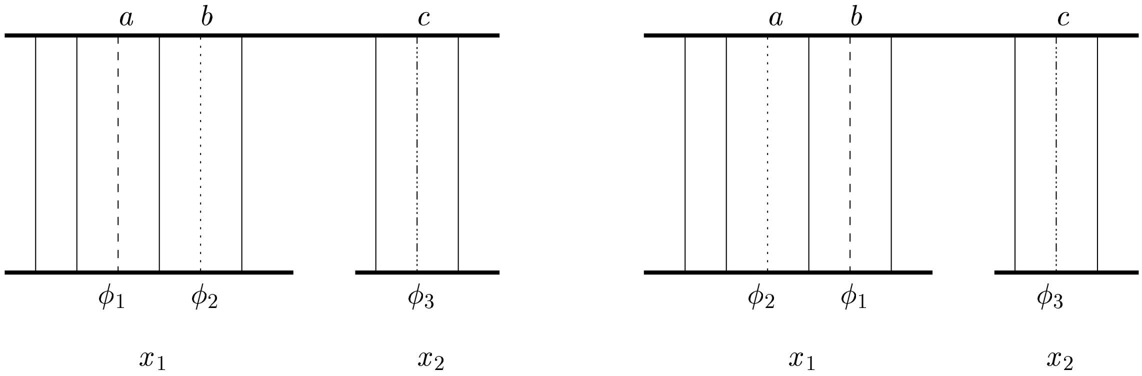

Figure 10: Free diagrams for G_3. This diagram gives rise to P.

order λ' contributions to C_{123} (but not the logarithms of x_{12}^2 in (4.23), which cannot appear in the x-independent mixing matrices).

Three-point correlator $G_3'(x_1, x_2)$. We now consider the second 3-point function, $G_3'(x_1, x_2)$, of eq. (4.7). Its structure is much the same as for $G_3(x_1, x_2)$ leading to the following expression

$$G_3'(x_1, x_2) = \frac{1 + \alpha/2(\gamma + 1 - \log 4\pi - s)}{J\sqrt{N^{J+3}}} \frac{N^{J+2}}{\sqrt{J_1 N^{J_1+2} N^{J_2+1}}} \times \\ \times \Delta(x_1)^{J_1+2} \Delta(x_2)^{J_2+1} (P - \lambda Q K(x_1, x_2)) , \tag{4.25}$$

where P and Q are the phase factors to be determined shortly.

P is the phase factor which we get by summing the contributions from the two free diagrams depicted in figure 10.

$$P = \sum_{a=1}^{J_1+1} \sum_{b=a+1}^{J_1+2} \sum_{c=J_1+3}^{J+3} \bar{\mathrm{q}}_2^{b-a-1} \bar{\mathrm{q}}_3^{c-a-2} + \sum_{a=1}^{J_1+1} \sum_{b=a+1}^{J_1+2} \sum_{c=J_1+3}^{J+3} \bar{\mathrm{q}}_2^{J-(b-a-1)} \bar{\mathrm{q}}_3^{c-b-1} \tag{4.26}$$

Converting the above sun into an integral one finds

$$\begin{aligned} P &= J^3 \int_0^{J_1/J} da \int_a^{J_1/J} db \int_{J_1/J}^1 dc\, e^{-2\pi i n_2 (b-a)} e^{-2\pi i n_3 (c-a)} + \\ &\quad + J^3 \int_0^{J_1/J} da \int_a^{J_1/J} db \int_{J_1/J}^1 dc\, e^{-2\pi i n_2 (-1)(b-a)} e^{-2\pi i n_3 (c-b)} \\ &= J^3 \int_0^{J_1/J} da \int_0^{J_1/J} db \int_{J_1/J}^1 dc\, e^{-2\pi i n_2 (b-a)} e^{-2\pi i n_3 (c-a)} . \end{aligned} \tag{4.27}$$

Evaluating the integral we finally arrive at

$$P = -J^3 \frac{\sin(\pi n_2 J_1/J) \sin(\pi n_3 J_1/J) \sin(\pi(n_3 + n_2) J_1/J)}{\pi^3 (n_3 + n_2) n_2 n_3} . \tag{4.28}$$

Now we have to account for the interacting diagrams. In the case where ϕ_1 takes part in the interaction we have four diagrams which are shown in figure 11. The corresponding

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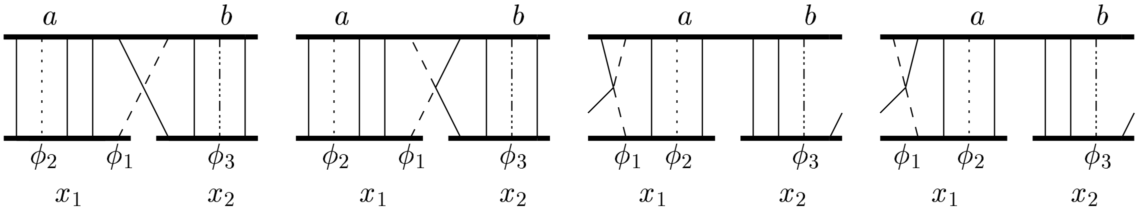

Figure 11: Interacting diagrams for G_3' contributing to Q_1. The ϕ_1 line is in the a^{th} position in the $\bar{O}^J_{n_2,n_3}(0)$ trace and ϕ_3 is in the b^{th} position.

phase factor is

$$\begin{aligned} Q_1 \,=\, &\sum_{a=1}^{J_1+1} \sum_{b=J_1+4}^{J+3} -\bar{\mathrm{q}}_2^{J-(J_1+3-a)}\bar{\mathrm{q}}_3^{b-(J_1+3)} + \bar{\mathrm{q}}_2^{J-(J_1+2-a)}\bar{\mathrm{q}}_3^{b-(J_1+2)} + \\ &+ \sum_{a=3}^{J_1+2} \sum_{b=J_1+3}^{J+2} \bar{\mathrm{q}}_2^{a-3}\bar{\mathrm{q}}_3^{b-4} - \bar{\mathrm{q}}_2^{a-2}\bar{\mathrm{q}}_3^{b-3}\,, \end{aligned} \tag{4.29}$$

which in the BMN limit is

$$\begin{aligned} Q_1 &= 2\pi i(n_2+n_3)J(1-e^{-2\pi i(n_3+n_2)(-1)J_1/J}) \int_0^{J_1/J} da \int_{J_1/J}^1 dbe^{-2\pi i n_2 a} e^{-2\pi i n_3 b} \\ &= -\frac{4}{\pi} J(n_3+n_2)^2 \frac{\sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{(n_3+n_2)n_2 n_3}\,. \end{aligned} \tag{4.30}$$

In the case where ϕ_2 takes part in the interaction we have again four diagrams which are shown in figure 12. The corresponding phase factor is

$$\begin{aligned} Q_2 \,=\, &\sum_{a=1}^{J_1+1} \sum_{b=J_1+4}^{J+3} -\bar{\mathrm{q}}_2^{J_1+2-a)}\bar{\mathrm{q}}_3^{b-a-2} + \bar{\mathrm{q}}_2^{J_1+-a)}\bar{\mathrm{q}}_3^{b-a-2} + \\ &+ \sum_{a=3}^{J_1+2} \sum_{b=J_1+3}^{J+2} \bar{\mathrm{q}}_2^{-(a-3)}\bar{\mathrm{q}}_3^{b-a-1} - \bar{\mathrm{q}}_2^{-(a-2)}\bar{\mathrm{q}}_3^{b-a-1}\,, \end{aligned} \tag{4.31}$$

which in the BMN limit becomes

$$\begin{aligned} Q_2 &= -J^2 \frac{2\pi i n_2}{J}(1-e^{-2\pi i n_2 J_1/J}) \int_0^{J_1/J} da \int_{J_1/J}^1 db\, e^{-2\pi i n_2(-a)} e^{-2\pi i n_3(b-a)} \\ &= -\frac{4}{\pi} J n_2^2 \frac{\sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{(n_3+n_2)n_2 n_3}\,. \end{aligned} \tag{4.32}$$

For interacting ϕ_3 there are eight diagrams. The first four are depicted in figure 13, and the other four are obtained by exchanging ϕ_1 with ϕ_2. The phase factor associated with the four diagrams of figure 13 in the BMN limit is

$$Q_3^{(1)} = J^2 \frac{2\pi i n_3}{J}(1-e^{-2\pi i n_3 J_1/J}) \int_0^{J_1/J} da \int_a^{J_1/J} db\, e^{-2\pi i n_2(b-a)} e^{-2\pi i n_3(-a)}\,. \tag{4.33}$$

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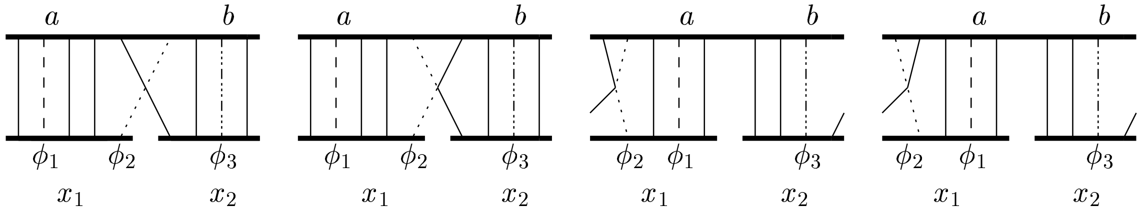

Figure 12: Interacting diagrams for G_3' contributing to Q_2.

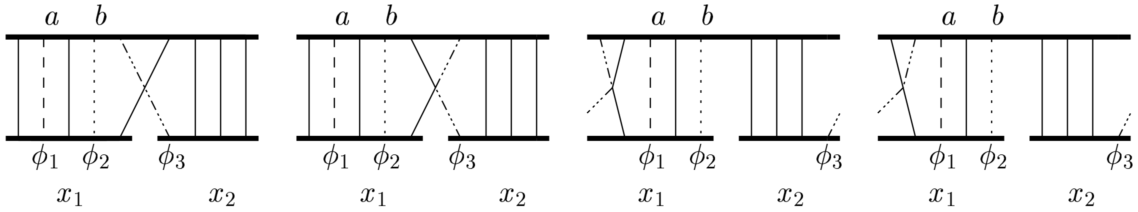

Figure 13: Interacting diagrams for G_3' contributing to Q_3. There are also diagrams with ϕ_1 and ϕ_2 exchanged.

The phase factor $Q_3^{(2)}$ for the remaining four diagrams is obtained by exchanging $n_1 \leftrightarrow n_2$ in $Q_3^{(1)}$. Adding the two we obtain

$$\begin{aligned} Q_3 &= Q_3^{(1)} + Q_3^{(2)} = 2\pi i n_3 J(1 - e^{-2\pi i n_3 J_1/J}) \int_0^{J_1/J} da \int_0^{J_1/J} db\, e^{-2\pi i n_2 (b-a)} e^{-2\pi i n_3(-a)} \\ &= -\frac{4}{\pi} J n_3^2 \, \frac{\sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{(n_3+n_2)n_2 n_3} . \end{aligned} \tag{4.34}$$

Taking everything into account, our final expression for G_3' takes the conformal form

$$G_3'(x_1,x_2) = C_{123}' \Delta(x_1)^{J_1+2+\alpha/2} \Delta(x_2)^{J_2+1+\alpha/2} \Delta(x_{12})^{-\alpha/2} \tag{4.35}$$

with the 3-point coefficient

$$C_{123}' = -\frac{J^2}{\sqrt{J_1} N \pi^3} \frac{\sin(\pi n_2 J_1/J)\sin(\pi n_3 J_1/J)\sin(\pi(n_3+n_2)J_1/J)}{(n_3+n_2)n_2 n_3} \left(1 - \frac{\alpha}{2}\right) . \tag{4.36}$$

This expression is again subtraction scheme independent. As in the case of G_3 discussed earlier, we will set $\alpha = 0$ on the right hand side of (4.36) to be safe from unknown mixing effects at order λ'.

4.2 Three-point functions with two string states

We are now ready to finally address the general case and calculate the 3-point coefficients of (4.1) and (4.2) for two non-chiral operators. Here the mixing of the known single-trace BMN operators with double-trace corrections is important as it does contribute to the conformal expression (1.2). However, our goal is not to derive the conformal expression on the right hand side of (1.2) (which must be correct anyway, as far as the mixing effects are such that we are dealing with scalar conformal primary operators). Our goal is to calculate the coefficient C_{123}. At leading order, the only mixing effect which contributes to the right

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hand side of (1.2) is the mixing with the double-traces of the barred operator $\bar{\mathcal{O}}^J_{n_1n_2n_3}(0)$ in (4.1) and (4.2). These mixing effects will affect the free-theory contribution $C^{\rm free}_{123}$ and also the logarithmic terms $\lambda' \log|x_1|$ and $\lambda' \log|x_2|$ due to interactions of the double-trace in $\bar{\mathcal{O}}^J_{n_1n_2n_3}(0)$ with the BMN operators at x_1 and x_2. But, these mixing effects cannot affect the third logarithm, $\lambda' \log|x_1 - x_2|$. Hence out programme is to assume the conformal form, and by carefully evaluating the terms proportional to $\lambda' \log|x_1 - x_2|$, to determine C_{123}. In doing so we can neglect the double-trace corrections and work with the original single-trace expressions.

We start with

$$G_3(x_1, x_2) = \langle \bar{\mathcal{O}}^J_{n_1n_2n_3}(0) \mathcal{O}^{J_1}_{n_1'n_2'n_3'}(x_1) \mathcal{O}^{J_2}_{\rm vac}(x_2) \rangle \,. \tag{4.37}$$

The calculation is done as in the previous subsection, except that now we have additional phase factors coming from non-zero n_2' and n_3'. The result for phase factor X coming from the free diagrams of figure 5 is obtained from (4.13) by substituting $n_2 - n_2'/y$ for n_2 and $n_3 - n_3'/y$ for n_3 where $y = J_1/J$. The final result is

$$X = J^3 2^3 \, \frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_2+n_3)y)}{(2\pi)^3 (n_2 + n_3 - \frac{n_3'+n_2'}{y})(n_2 - \frac{n_2'}{y})(n_3 - \frac{n_3'}{y})} \,. \tag{4.38}$$

To simplify notation somewhat, we will define

$$\Pi := \frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_2+n_3)y)}{(n_2 + n_3 - \frac{n_3'+n_2'}{y})(n_2 - \frac{n_2'}{y})(n_3 - \frac{n_3'}{y})} \,. \tag{4.39}$$

Next we evaluate the interacting diagrams of figure 6,

$$\begin{aligned}
Y_{123} \;=\; & \sum_{a=1}^{J_1+1} \sum_{b=a+1}^{J_1+2} [-\bar{q}_2^{\,J-(J_1+3-a-1)} \bar{q}_3^{\,J-(J_1+3-b)} + \bar{q}_2^{\,J-(J_1+3-a-2)} \bar{q}_3^{\,J-(J_1+3-b-1)}] r_2^{a-1} r_3^{b-2} + \\
& + \sum_{a=3}^{J_1+2} \sum_{b=a+1}^{J_1+3} [\bar{q}_2^{a-3} \bar{q}_3^{b-4} - \bar{q}_2^{a-2} \bar{q}_3^{b-3}] r_2^{a-3} r_3^{b-4} \\
= & \sum_{a=1}^{J_1+1} \sum_{b=a+1}^{J_1+2} \bar{q}_2^{\,-(J_1-a+1)} \bar{q}_3^{\,-(J_1-b+2)} r_2^{a-1} r_3^{b-2} (1 - \bar{q}_2^{-1} \bar{q}_3^{-1}) + \\
& + \sum_{a=3}^{J_1+2} \sum_{b=a+1}^{J_1+3} \bar{q}_2^{a-3} \bar{q}_3^{b-4} r_2^{a-3} r_3^{b-4} (1 - \bar{q}_2 \bar{q}_3) \to \\
& \to \frac{2\pi i(n_2+n_3)}{J} \Bigg[\sum_{a=3}^{J_1+2} \sum_{b=a+1}^{J_1+3} (\bar{q}_2 r_2)^{a-3} (\bar{q}_3 r_3)^{b-4} - \\
& \qquad - \sum_{a=1}^{J_1+1} \sum_{b=a+1}^{J_1+2} (\bar{q}_2 r_2)^{a-1} (\bar{q}_3 r_3)^{b-2} \bar{q}_2^{-J_1} \bar{q}_3^{-J_1} \Bigg] \,.
\end{aligned} \tag{4.40}$$

Converting the last sum into an integral we obtain,

$$Y_{123} \;=\; 2\pi i(n_2+n_3) J \int_0^y da \int_a^y db \, e^{-2\pi i(n_2 - \frac{n_2'}{y})a} e^{-2\pi i(n_3 - \frac{n_3'}{y})b} [1 - e^{2\pi i(n_3+n_2)y}]$$

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$$= 2\pi i J(n_2+n_3)[1-e^{2\pi i(n_3+n_2)y}]\frac{e^{(A_3+A_2)y}A_3 - e^{A_3 y}A_2 - e^{A_3 y}A_3 + A_2}{(A_3+A_2)A_3A_2}\,,$$

where $A_2 = -2\pi i(n_2 - n_2'/y)$, $A_3 = -2\pi i(n_3 - n_3'/y)$, $r_2 = e^{2\pi i n_2'/J_1}$ and $r_3 = e^{2\pi i n_3'/J_1}$. Expression for Y_{132} is obtained by interchanging A_2 and A_3 in Y_{123}. After some algebra we get

$$Y_{123}+Y_{132} = \frac{4(n_3+n_2)(n_3+n_2-\frac{n_2'+n_3'}{y})J}{\pi}\,\Pi\,. \tag{4.41}$$

Similarly, for diagrams of figure 7 we obtain (in the BMN limit)

$$\begin{aligned} Y_{231} &= 2\pi i n_2 J\int_0^y da\int_0^y db\, e^{2\pi i(n_2-\frac{n_2'}{y})b}e^{2\pi i(n_3-\frac{n_3'}{y})(b-a)}[1-e^{-2\pi i n_2 y}] \\ &= 2\pi i n_2 J[1-e^{-2\pi i n_2 y}]\frac{e^{-A_2 y}A_2 + e^{-A_2 y}A_3 - e^{-(A_3+A_2)y}A_2 - A_3}{(A_3+A_2)A_3A_2}\,. \end{aligned} \tag{4.42}$$

Now consider the above diagrams with ϕ_1 and ϕ_3 exchanged. In the BMN limit we find

$$\begin{aligned} Y_{213} &= -2\pi i n_2 J\int_0^y da\int_0^y db\, e^{-2\pi i(n_2-\frac{n_2'}{y})(-a)}e^{-2\pi i(n_3-\frac{n_3'}{y})(b-a)}[1-e^{-2\pi i n_2 y}] \\ &= -2\pi i n_2 J[1-e^{-2\pi i n_2 y}]\frac{e^{-A_2 y}A_3 + e^{A_3 y}A_2 - A_2 - A_3}{(A_3+A_2)A_3A_2}\,. \end{aligned} \tag{4.43}$$

Summing the two we arrive at

$$Y_{231}+Y_{213} = \frac{4n_2(n_2-\frac{n_2'}{y})J}{\pi}\,\Pi\,. \tag{4.44}$$

Similarly for the diagrams of figure 8 we get:

$$Y_{312}+Y_{321} = \frac{4n_3(n_3-\frac{n_3'}{y})J}{\pi}\,\Pi\,. \tag{4.45}$$

Using the above phase factors and concentrating on the logarithmic terms of the 3-point function we have

$$\begin{aligned} G_3(x_1,x_2) = C_{123}^{(0)}\,\Delta(x_1)^{J_1+3}\Delta(x_2)^{J_2}\Bigg[1 - \frac{\lambda'}{4}\Bigg(n_2\left(n_2-\frac{n_2'}{y}\right) + n_3\left(n_3-\frac{n_3'}{y}\right) + \\ + (n_3+n_2)\left(n_3+n_2-\frac{n_2'+n_3'}{y}\right)\Bigg)\times \\ \times\log\frac{x_1^2x_2^2}{x_{12}^2} + +C'\log x_1^2 + \cdots\Bigg]\,. \end{aligned} \tag{4.46}$$

The last term in the equation above comes from the diagrams of figure 9 which no longer sum to zero as in the case with two supergravity states. However we do not need to know the coefficient of this term since the $\log x_1^2$ receives corrections from the the double-trace operators. From the equation above one can easily read the coefficient C_{123} at order g_2

$$C_{123} = C_{123}^{(0)}\,\frac{n_2(n_2-\frac{n_2'}{y}) + n_3(n_3-\frac{n_3'}{y}) + (n_3+n_2)(n_3+n_2-\frac{n_2'+n_3'}{y})}{n_2^2+n_3^2+(n_2+n_3)^2 - \frac{n_2'^2+n_3'^2+(n_2'+n_3')^2}{y^2}}\,, \tag{4.47}$$

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where

$$C_{123}^{(0)} = \frac{J\sqrt{J}\sqrt{1-y}}{Ny\pi^3} \frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_2+n_3)y)}{(n_2+n_3-\frac{n_3'+n_2'}{y})(n_2-\frac{n_2'}{y})(n_3-\frac{n_3'}{y})}\,. \tag{4.48}$$

Three-point correlator $G_3'(x_1,x_2)$. Finally we consider the G_3' function with two string states

$$G_3'(x_1,x_2) = \langle \bar{\mathcal{O}}^J_{n_1 n_2 n_3}(0)\mathcal{O}^{J_1}_{n_2', -n_2'}(x_1)\mathcal{O}^{J_2}_0(x_2)\rangle\,. \tag{4.49}$$

Similarly to the earlier analysis, we determine P from free diagrams in figure 10,

$$P = -\frac{J^3\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_3+n_2)y)}{\pi^3(n_3+n_2-\frac{n_2'}{y})(n_2-\frac{n_2'}{y})n_3}\,. \tag{4.50}$$

The diagrams of figure 11, figure 12 and figure 12 lead to the expressions for Q_1, Q_2 and Q_3 respectively

$$\begin{aligned}
Q_1 &= -\frac{4}{\pi}J(n_3+n_2)\left(n_3+n_2-\frac{n_2'}{y}\right)\frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_3+n_2)y)}{(n_3+n_2-n_2'/y)(n_2-n_2'/y)n_3}\\
Q_2 &= -\frac{4}{\pi}Jn_2\left(n_2-\frac{n_2'}{y}\right)\frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_3+n_2)y)}{(n_3+n_2-n_2'/y)(n_2-n_2'/y)n_3}\\
Q_3 &= -\frac{4}{\pi}Jn_3^2\,\frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_3+n_2)y)}{(n_3+n_2-n_2'/y)(n_2-n_2'/y)n_3}\,.
\end{aligned} \tag{4.51}$$

Using the above results the three-point function reads

$$\begin{aligned}
G_3'(x_1,x_2) &= \tilde{C}_{123}^{(0)}\ \Delta(x_1)^{J_1+2}\Delta(x_2)^{J_2+1}\times\\
&\times\left[1-\frac{\lambda'}{4}\left(n_3^2+n_2\left(n_2-\frac{n_2'}{y}\right)+\left(n_3+n_2)(n_3+n_2-\frac{n_2'}{y}\right)\right)\times\right.\\
&\left.\qquad\times\log\frac{x_1^2x_2^2}{x_{12}^2}+C'\log x_1^2+C''\log x_2^2+\cdots\right],
\end{aligned}$$

where

$$\tilde{C}_{123}^{(0)} = -\frac{J\sqrt{J}}{\sqrt{y}N\pi^3}\frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_3+n_2)y)}{(n_3+n_2-n_2'/y)(n_2-n_2'/y)n_3}\,. \tag{4.52}$$

And our final expression for $\tilde{C}_{123}$ is

$$\tilde{C}_{123} = \tilde{C}_{123}^{(0)}\frac{n_3^2+n_2(n_2-n_2'/y)+(n_3+n_2)(n_3+n_2-n_2'/y)}{n_2^2+n_3^2+(n_2+n_3)^2-2n_2'^2/x^2}\,. \tag{4.53}$$

5. Tests of the correspondence

In this section we test the correspondence proposed in [6] and outlined in section 2 against the gauge theory results of section 4.

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5.1 $\mathbf{1_{-(n_2'+n_3')}2_{n_2'}3_{n_3'}+vac\to 1_{-(n_2+n_3)}2_{n_2}3_{n_3}}$

In this case the external string state is

$$\langle\Phi| = \langle 0|\alpha^{3i_1}_{-(n_2+n_3)}\alpha^{3i_2}_{n_2}\alpha^{3i_3}_{n_3}\alpha^{1i_1}_{-(n_2'+n_3')}\alpha^{1i_2}_{n_2'}\alpha^{1i_3}_{n_3'}\,. \tag{5.1}$$

In the expression above, 1 and 3 denote the first and the third string in the vertex. In order to avoid confusion, and distinguish from the indices corresponding to scalar impurites, ϕ_1, ϕ_2, ϕ_3, we label the latter with i_1, i_2, i_3. Since all three SYM impurities are different, we note that $i_1\neq i_2\neq i_3$ and the repeated indices in the external states are not summed over.

The contribution of the first part of the prefactor P_I is

$$\begin{aligned}\langle\Phi|\mathsf{P}_I|V_B\rangle \;=\; &\frac{1}{2\mu}\left[\frac{n_2'^2+n_3'^2+(n_2'+n_3')^2}{y^2}-{n_2}^2-{n_3}^2-(n_2+n_3)^2\right]\times\\ &\times\hat N^{31}_{n_2,n_2'}\hat N^{31}_{n_3,n_3'}\hat N^{31}_{n_2+n_3,n_2'+n_3'}\end{aligned} \tag{5.2}$$

which, using the expression for the Neumann matrices (see appendix),

$$\hat N^{31}_{mn} = \hat N^{31}_{-m-n} = \frac{(-1)^{m+n+1}}{\pi}\frac{\sin(\pi m y)}{\sqrt{y}(m-n/y)}+\mathcal{O}\left(\frac{1}{\mu^2}\right), \tag{5.3}$$

becomes

$$\langle\Phi|\mathsf{P}_I|V_B\rangle = -\frac{1}{2\mu y^{3/2}\pi^3}\left[\frac{n_2'^2+n_3'^2+(n_2'+n_3')^2}{y^2}-n_2^2-n_3^2-(n_2+n_3)^2\right]\,\Pi\,. \tag{5.4}$$

Now consider P_{II}. The contributing terms in P_{II} for the state under consideration are

$$\begin{aligned}\mathsf{P}_{II} \;=\; &\frac{1}{2}\Bigg[\left(\frac{\omega_{3n_2}}{\alpha_3}+\frac{\omega_{1n_2'}}{\alpha_1}\right)\left(\hat N^{31}_{n_2,-n_2'}-\hat N^{31}_{n_2,n_2'}\right)\alpha^{3i_2\dagger}_{n_2}\alpha^{1i_2\dagger}_{n_2'}+\\ &+\left(\frac{\omega_{3n_3}}{\alpha_3}+\frac{\omega_{1n_3'}}{\alpha_1}\right)\left(\hat N^{31}_{n_3,-n_3'}-\hat N^{31}_{n_3,n_3'}\right)\alpha^{3i_3\dagger}_{n_3}\alpha^{1i_3\dagger}_{n_3'}+\\ &+\left(\frac{\omega_{3n_1}}{\alpha_3}+\frac{\omega_{1n_1'}}{\alpha_1}\right)\left(\hat N^{31}_{-n_1,n_1'}-\hat N^{31}_{-n_1,-n_1'}\right)\alpha^{3i_1\dagger}_{n_1}\alpha^{1i_1\dagger}_{n_1'}\Bigg]\,.\end{aligned} \tag{5.5}$$

Using this we find

$$\begin{aligned}\langle\Phi|\mathsf{P}_{II}|V_B\rangle \;=\; &-\frac{1}{4\mu}\Bigg[\left(n_2^2-\frac{n_2'^2}{y^2}\right)N^{31}_{-n_2,-n_2'}\hat N^{31}_{n_3,n_3'}\hat N^{31}_{-n_1,-n_1'}+\\ &+\left(n_3^2-\frac{n_3'^2}{y^2}\right)N^{31}_{-n_3,-n_3'}\hat N^{31}_{n_2,n_2'}\hat N^{31}_{-n_1,-n_1'}+\\ &+\left(n_1^2-\frac{n_1'^2}{y^2}\right)N^{31}_{n_1,n_1'}\hat N^{31}_{n_3,n_3'}\hat N^{31}_{n_2,n_2'}\Bigg]\,.\end{aligned} \tag{5.6}$$

Recalling that $n_1=-(n_2+n_3)$ and $n_1'=-(n_2'+n_3')$ and substituting into the above result the expressions (5.3) for the Neumann matrices $\hat N$ and expressions for the Neumann matrices $N_{-,-}$ (see appendix)

$$N^{31}_{-m-n} = \frac{2(-1)^{m+n}}{\pi}\frac{n\sin(\pi m y)}{y^{3/2}(m^2-n^2/y^2)}+\mathcal{O}\left(\frac{1}{\mu^2}\right), \tag{5.7}$$

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we obtain

$$\langle\Phi|P_{II}\hat{V}|0\rangle = -\frac{1}{2\mu\pi^3 y^{5/2}}\left[n_2'\left(n_2-\frac{n_2'}{y}\right)+n_3'\left(n_3-\frac{n_3'}{y}\right)+n_1'\left(n_1-\frac{n_1'}{y}\right)\right]\,\Pi\,. \quad (5.8)$$

Adding these results for P_I and P_{II} and multiplying by $(-1)^p C_{norm} = -g_2\frac{\sqrt{y(1-y)}}{\sqrt{J}}$ we get the string theory answer

$$-\frac{g_2}{2\mu}\frac{\sqrt{1-y}}{\pi^3\sqrt{J}\,y}\left[n_2\left(n_2-\frac{n_2'}{y}\right)+n_3\left(n_3-\frac{n_3'}{y}\right)+\left(n_3+n_2)(n_3+n_2-\frac{n_2'+n_3'}{y}\right)\right]\,\Pi \quad (5.9)$$

which is *exactly the SYM result*, $\mu(\Delta_1+\Delta_2-\Delta_3)C_{123}$, since

$$\mu(\Delta_1+\Delta_2-\Delta_3) = \frac{1}{2\mu}\left[\frac{n_2'^2+n_3'^2+(n_2'+n_3')^2}{y^2}-n_2^2-n_3^2-(n_2+n_3)^2\right] \quad (5.10)$$

and C_{123}, given by (4.47), (4.48), reads

$$C_{123} = \frac{J\sqrt{J}\sqrt{1-y}}{Ny\pi^3}\,\frac{n_2(n_2-\frac{n_2'}{y})+n_3(n_3-\frac{n_3'}{y})+(n_3+n_2)(n_3+n_2-\frac{n_2'+n_3'}{y})}{n_2^2+n_3^2+(n_2+n_3)^2-\frac{n_2'^2+n_3'^2+(n_2'+n_3')^2}{y^2}}\,\Pi\,. \quad (5.11)$$

5.2 $\mathbf{1}_{n_2'}\mathbf{2}_{-(n_2'+n_3')}\mathbf{3}_{n_3'} + \mathbf{vac} \to \mathbf{1}_{-(n_2+n_3)}\mathbf{2}_{n_2}\mathbf{3}_{n_3}$

In this case the external string state is

$$\langle\Phi| = \langle 0|\alpha^{3i_1}_{-(n_2+n_3)}\alpha^{3i_2}_{n_2}\alpha^{3i_3}_{n_3}\alpha^{1i_1}_{n_2'}\alpha^{1i_2}_{-(n_2'+n_3')}\alpha^{1i_3}_{n_3'} \quad (5.12)$$

and the operator $\mathcal{O}_1(x_1)$ is

$$\mathcal{O}_1(x_1) = \frac{1}{J\sqrt{N^{J+3}}}\sum_{0\le a,b}^{a+b\le J}\left[r_2^a r_3^{a+b}\,\mathrm{tr}(\phi_2 Z^a\phi_1 Z^b\phi_3 Z^{J-a-b}) + \right.$$
$$\left. + r_3^a r_2^{a+b}\,\mathrm{tr}(\phi_2 Z^a\phi_3 Z^b\phi_1 Z^{J-a-b})\right]. \quad (5.13)$$

Using the cyclic property of the trace, $\mathcal{O}_1(x_1)$ can also be written as

$$\mathcal{O}_1(x_1) = \frac{1}{J\sqrt{N^{J+3}}}\sum_{0\le a,b}^{a+b\le J}\left[\tilde{r_2}^a\tilde{r_3}^{a+b}\,\mathrm{tr}(\phi_1 Z^a\phi_2 Z^b\phi_3 Z^{J-a-b}) + \right.$$
$$\left. + \tilde{r_3}^a\tilde{r_2}^{a+b}\,\mathrm{tr}(\phi_1 Z^a\phi_3 Z^b\phi_2 Z^{J-a-b})\right], \quad (5.14)$$

where $\tilde{r_2} = (r_2 r_3)^{-1}$ and $\tilde{r_3} = r_3$. Hence, we find the 3-point function for this process by substituting $n_2' \to -(n_2'+n_3')$, $n_3' \to n_3'$ and $-(n_2'+n_3') = n_1 \to n_2'$ into the SYM expression (5.11) and also to (5.10).

Thus, the result on the gauge theory side for this process is

$$\mu(\Delta_1+\Delta_2-\Delta_3)C_{123} = -\frac{g_2}{2\mu}\frac{\sqrt{1-y}}{\pi^3\sqrt{J}\,y}\frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_2+n_3)y)}{(n_2+\frac{n_2'+n_3'}{y})(n_3-\frac{n_3'}{y})(n_2+n_3+\frac{n_2'}{y})}\times$$
$$\times\left[n_2\left(n_2+\frac{n_2'+n_3'}{y}\right)+n_3\left(n_3-\frac{n_3'}{y}\right)+\right.$$
$$\left.+(n_2+n_3)\left(n_2+n_3+\frac{n_2'}{y}\right)\right]. \quad (5.15)$$

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On the string theory side of the correspondence the calculation follows the same lines as in **5.1**, and the result is in precise agreement with the SYM formula (5.15).

This is also true for the remaining four cases involving different permutations of the three ϕ's in the trace of the shortest BMN operator.

5.3 $\mathbf{1}_{-\mathbf{n}_2'}\mathbf{2}_{\mathbf{n}_2'}+\mathbf{3}_0 \to \mathbf{1}_{-(\mathbf{n}_2+\mathbf{n}_3)}\mathbf{2}_{\mathbf{n}_2}\mathbf{3}_{\mathbf{n}_3}$

Here we consider the case which corresponds to G_3', i.e. where instead of the vacuum state for the string 2, we have a supergravity state. The external string state now is

$$\langle\Phi| = \langle 0|\alpha^{3i_1}_{-(n_2+n_3)}\alpha^{3i_2}_{n_2}\alpha^{3i_3}_{n_3}\alpha^{1i_1}_{-n_2'}\alpha^{1i_2}_{+n_2'}\alpha^{2i_3}_{0}\,. \tag{5.16}$$

The result in the SYM side can be obtained from (4.53) and (4.52) and is

$$\begin{aligned}\mu(\Delta_1+\Delta_2-\Delta_3)\tilde{C}_{123} &= \frac{1}{2\mu}\frac{J\sqrt{J}}{\sqrt{y}N\pi^3}\frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_3+n_2)y)}{(n_3+n_2-n_2'/y)(n_2-n_2'/y)n_3}\times\\ &\quad\times\left(n_3^2+n_2\left(n_2-\frac{n_2'}{y}\right)+(n_3+n_2)\left(n_3+n_2-\frac{n_2'}{y}\right)\right).\end{aligned} \tag{5.17}$$

This expression is again, as it is easy to check using the Neumann matrices from the appendix, in precise agreement with the expression on the string theory side of the duality relation (2.1).

5.4 $\mathbf{1}_{\mathbf{n}_2'}\mathbf{2}_{-\mathbf{n}_2'}+\mathbf{3}_0 \to \mathbf{1}_{-(\mathbf{n}_2+\mathbf{n}_3)}\mathbf{2}_{\mathbf{n}_2}\mathbf{3}_{\mathbf{n}_3}$

As a final example we consider the case where the external string state is

$$\langle\Phi| = \langle 0|\alpha^{3i_1}_{-(n_2+n_3)}\alpha^{3i_2}_{n_2}\alpha^{3i_3}_{n_3}\alpha^{1i_1}_{n_2'}\alpha^{1i_2}_{-n_2'}\alpha^{2i_3}_{0}\,. \tag{5.18}$$

The result on the SYM side is given by

$$\begin{aligned}\mu(\Delta_1+\Delta_2-\Delta_3)\tilde{C}_{123} &= \frac{1}{2\mu}\frac{J\sqrt{J}}{\sqrt{y}N\pi^3}\frac{\sin(\pi n_2 y)\sin(\pi n_3 y)\sin(\pi(n_3+n_2)y)}{(n_3+n_2+n_2'/y)(n_2+n_2'/y)n_3}\times\\ &\quad\left(n_3^2+n_2\left(n_2+\frac{n_2'}{y}\right)+(n_3+n_2)\left(n_3+n_2+\frac{n_2'}{y}\right)\right)\end{aligned} \tag{5.19}$$

which is in precise agreement with the string vertex-correlator duality prediction (2.1).

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A. Some useful expressions in string field theory

Here we outline the pp-wave string field theory conventions used in the text. The combination $\alpha' p^+$ for the r-th string is denoted $\alpha_{\rm r}$ and $\sum_{\rm r=1}^{3} \alpha_{\rm r} = 0$. As is standard in the literature, we will choose a frame in which $\alpha_3 = -1$

$$\alpha_{\rm r} = \alpha' p^+_{(\rm r)} : \qquad \alpha_3 = -1 , \qquad \alpha_1 = y , \qquad \alpha_2 = 1 - y . \tag{A.1}$$

In terms of the U(1) R-charges of the BMN operators in the three-point function, $\langle \mathcal{O}_1^{J_1} \mathcal{O}_2^{J_2} \bar{\mathcal{O}}_3^{J} \rangle$, where $J = J_1 + J_2$, we have

$$y = \frac{J_1}{J} , \qquad 1 - y = \frac{J_2}{J} , \qquad 0 < y < 1 . \tag{A.2}$$

The effective SYM coupling constant (1.5) in the frame (A.1) takes a simple form

$$\lambda' = \frac{1}{(\mu p^+ \alpha')^2} \equiv \frac{1}{(\mu \alpha_3)^2} = \frac{1}{\mu^2} . \tag{A.3}$$

Here μ is the mass parameter which appears in the pp-wave metric, in the chosen frame it is dimensionless[6] and the expansion in powers of $1/\mu^2$ is equivalent to the perturbative expansion in λ'. Finally the frequencies are defined via,

$$\omega_{{\rm r}m} = \sqrt{m^2 + (\mu \alpha_{\rm r})^2} . \tag{A.4}$$

The infinite-dimensional Neumann matrices, $N^{\rm rs}_{mn}$ are usually specified in the original a-oscillator basis of the string field theory. In this basis the bosonic overlap factor $|V_B\rangle$ of the 3-strings vertex is given by

$$|V_B\rangle = \exp\left(\frac{1}{2} \sum_{\rm r,s=1}^{3} \sum_{m,n=-\infty}^{\infty} a^{{\rm r}\, I\dagger}_m N^{\rm rs}_{mn} a^{{\rm s}\, I\dagger}_n \right) |0\rangle . \tag{A.5}$$

However, for the purposes of the pp-wave/SYM correspondence it is more convenient to use another, the so-called BMN or α-basis of string oscillators, which is in direct correspondence with the BMN operators in gauge theory. The two bases are related as follows:

$$\alpha_n = \frac{1}{\sqrt{2}} (a_{|n|} - i\, {\rm sign}(n) a_{-|n|}) , \qquad \alpha_0 = a_0 , \tag{A.6}$$

and satisfy the same oscillator algebra

$$[\alpha_m, \alpha_n^\dagger] = \delta_{mn} . \tag{A.7}$$

In this basis, the bosonic overlap factor (A.5) in the vertex reads

$$|V_B\rangle = \exp\left(\frac{1}{2} \sum_{\rm r,s=1}^{3} \sum_{m,n=-\infty}^{\infty} \alpha^{{\rm r}\, I\dagger}_m \hat{N}^{\rm rs}_{mn} \alpha^{{\rm s}\, I\dagger}_n \right) , \tag{A.8}$$

[6] It is $p^+\mu$ which is invariant under longitudinal boosts and is frame-independent.

where $\hat{N}$ are the Neumann matrices in the α-basis and are related to the N's via (here $m, n > 0$):

$$
\begin{aligned}
\hat{N}^{\mathrm{rs}}_{mn} &= \hat{N}^{\mathrm{rs}}_{-m-n} := \frac{1}{2}(N^{\mathrm{rs}}_{mn} - N^{\mathrm{rs}}_{-m-n}) \,, \\
\hat{N}^{\mathrm{rs}}_{m-n} &= \hat{N}^{\mathrm{rs}}_{-mn} := \frac{1}{2}(N^{\mathrm{rs}}_{mn} + N^{\mathrm{rs}}_{-m-n}) \,, \\
\hat{N}^{\mathrm{rs}}_{m0} &= \hat{N}^{\mathrm{rs}}_{-m0} := \frac{1}{\sqrt{2}} N^{\mathrm{rs}}_{m0} = \hat{N}^{\mathrm{rs}}_{0m} = \hat{N}^{\mathrm{rs}}_{0-m}, \qquad \text{(A.9)} \\
\hat{N}^{\mathrm{rs}}_{00} &:= N^{\mathrm{rs}}_{00} \,. \qquad \text{(A.10)}
\end{aligned}
$$

We now copy the explicit expressions for the Neumann matrices [23] in the a-basis from the [6, appendix]. These expressions are needed for our calculations in section 5.

$$
\begin{aligned}
N^{31}_{mn} &= \frac{2(-1)^{m+n+1}}{\pi} \frac{m \sin(\pi m y)}{\sqrt{y}(m^2 - n^2/y^2)} + \mathcal{O}\left(\frac{1}{\mu^2}\right), \\
N^{32}_{mn} &= \frac{2(-1)^{m}}{\pi} \frac{m \sin(\pi m y)}{\sqrt{1-y}(m^2 - n^2/(1-y)^2)} + \mathcal{O}\left(\frac{1}{\mu^2}\right), \\
N^{21}_{mn} &= \frac{1}{\mu} \frac{(-1)^{n+1}}{2\pi} \frac{1}{\sqrt{y(1-y)}} + \mathcal{O}\left(\frac{1}{\mu^3}\right), \\
N^{33}_{mn} &= \mathcal{O}\left(\frac{1}{\mu^3}\right), \\
N^{11}_{mn} &= \frac{1}{\mu} \frac{(-1)^{m+n}}{2\pi} \frac{1}{y} + \mathcal{O}\left(\frac{1}{\mu^3}\right), \\
N^{22}_{mn} &= \frac{1}{\mu} \frac{1}{2\pi} \frac{1}{1-y} + \mathcal{O}\left(\frac{1}{\mu^3}\right). \qquad \text{(A.11)}
\end{aligned}
$$

$$
\begin{aligned}
N^{31}_{-m-n} &= \frac{2(-1)^{m+n}}{\pi} \frac{n \sin(\pi m y)}{y^{3/2}(m^2 - n^2/y^2)} + \mathcal{O}\left(\frac{1}{\mu^2}\right), \\
N^{32}_{-m-n} &= \frac{2(-1)^{m+1}}{\pi} \frac{n \sin(\pi m y)}{(1-y)^{3/2}(m^2 - n^2/(1-y)^2)} + \mathcal{O}\left(\frac{1}{\mu^2}\right), \\
N^{21}_{-m-n} &= \mathcal{O}\left(\frac{1}{\mu^3}\right), \\
N^{33}_{-m-n} &= \frac{1}{\mu} \frac{2(-1)^{m+n}}{\pi} \sin(\pi m y) \sin(\pi n y) + \mathcal{O}\left(\frac{1}{\mu^3}\right), \\
N^{11}_{-m-n} &= \mathcal{O}\left(\frac{1}{\mu^3}\right), \\
N^{22}_{-m-n} &= \mathcal{O}\left(\frac{1}{\mu^3}\right). \qquad \text{(A.12)}
\end{aligned}
$$

$$
\begin{aligned}
N^{33}_{00} &= 0 \,, \qquad N^{31}_{00} = -\sqrt{y} \,, \qquad N^{32}_{00} = -\sqrt{1-y} \,, \\
N^{12}_{00} &= \frac{1}{\mu} \frac{(-1)}{4\pi} \frac{1}{\sqrt{y(1-y)}} = N^{21}_{00} \,, \\
N^{11}_{00} &= \frac{1}{\mu} \frac{1}{4\pi} \frac{1}{y} \,, \\
N^{22}_{00} &= \frac{1}{\mu} \frac{1}{4\pi} \frac{1}{1-y} \,. \qquad \text{(A.13)}
\end{aligned}
$$

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For the zero-positive Neumann matrices below we have

$$
\begin{aligned}
N^{31}_{0n} &= 0\,, \qquad N^{32}_{0n} = 0\,, \qquad N^{33}_{0n} = 0.\\
N^{13}_{0n} &= \frac{\sqrt{2}(-1)^{n+1}}{\pi}\frac{\sin(\pi n y)}{n\sqrt{y}} + \mathcal{O}\left(\frac{1}{\mu^2}\right),\\
N^{23}_{0n} &= \frac{\sqrt{2}(-1)^{n}}{\pi}\frac{\sin(\pi n y)}{n\sqrt{1-y}} + \mathcal{O}\left(\frac{1}{\mu^2}\right),\\
N^{21}_{0n} &= \frac{1}{\mu}\,\frac{\sqrt{2}(-1)^{n+1}}{4\pi}\frac{1}{\sqrt{y(1-y)}} + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{12}_{0n} &= -\frac{1}{\mu}\,\frac{\sqrt{2}}{4\pi}\frac{1}{\sqrt{y(1-y)}} + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{11}_{0n} &= \frac{1}{\mu}\,\frac{\sqrt{2}(-1)^{n}}{4\pi}\frac{1}{y} + \mathcal{O}\left(\frac{1}{\mu^3}\right),\\
N^{22}_{0n} &= \frac{1}{\mu}\,\frac{\sqrt{2}}{4\pi}\frac{1}{1-y} + \mathcal{O}\left(\frac{1}{\mu^3}\right).
\end{aligned}
\tag{A.14}
$$

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